Assignment 4

Learning Objective:

Determine areas under curves through integration and apply integration to work problems.

4-1. Which of the following statements regarding integration is FALSE?

1. It is the inverse of addition
2. It is the direct opposite of differentiation
3. $F(x) = \int f(x)\ dx$
4. The derivative of a function is given and the function must be found

4-2. In the integral $\int f(x)\ dx$, dx is known as the

1. limit of integration
2. integral sign
3. differential
4. integrand

4-3. If $y = \int x\ dx$, then x is known as the

1. integrand
2. differential
3. integral sign
4. limit of integration

4-4. An integral is used only to represent an area under a curve.

1. True
2. False

● Another way of viewing the expression

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k)\, \Delta x$$

is to substitute the value of the x coordinate into the function $f(x_k)$ to find the corresponding y coordinate. Hence, for k = 1 to n, $y_0 = f(x_0)$, $y_1 = f(x_1)$, $y_2 = f(x_2)$, ..., $y_n = f(x_n)$.

Therefore, the sum of the areas of the rectangles with width Δx and heights y_0, y_1, y_2, ..., y_n may be written as

$$A = \lim_{n \to \infty} (y_0 \Delta x + y_1 \Delta x + y_2 \Delta x + \ldots + y_n \Delta x)$$

The limit of this sum of products as $n \to \infty$ equals the integral from a to b of y dx, or
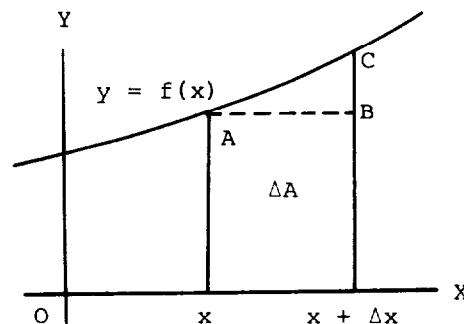
$$\int_a^b y\ dx$$

Figure 4A.--Area ΔA.

IN ANSWERING ITEM 4-5, REFER TO FIGURE 4A.

4-5. Under what condition can ΔA closely approximate the area under the curve y = f(x) between points x and x + Δx?

1. When Δx is extremely small
2. When f(x) equals f(x + Δx)
3. When point B moves to the right
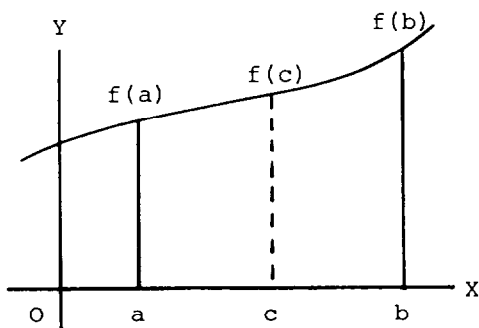4. When Δx increase without limit

Figure 4B.--Intermediate value between
a and b.

IN ANSWERING ITEM 4-6, REFER TO
FIGURE 4B.

4-6. The Intermediate Value Theorem
actually measures the area of a
rectangle with width b - a and
intermediate height

1. $\dfrac{f(b) - f(a)}{2}$

2. $f(b) - f(a)$

3. $f(a) + f(b)$

4. $f(c)$

● When calculating the area under a
curve, you must ensure the curve,
for example $y = f(x)$, is continuous at
all points over which the area is being
computed. If the curve is not continuous,
the area cannot be calculated by direct
methods.

4-7. If $_cA_x = F(x) - F(c)$ and $x = n$,
then the area between points c
and n on a curve equals

1. $\dfrac{F(n) + F(c)}{2}$

2. $F(n) - F(c)$

3. $F(n) + F(c)$

4. $\dfrac{F(n) - F(c)}{2}$

4-8. Which of the following is
equivalent to $\dfrac{d}{dx} \int g(x)\ dx$?

1. $\dfrac{d}{dx} g(x)$

2. $g(x)$

3. $\dfrac{d}{dx} G(x)$

4. Both 2 and 3 above

4-9. $\dfrac{d}{dx}(x^4)$, $\dfrac{d}{dx}(x^4 + 5)$, and $\dfrac{d}{dx}(x^4 - 8)$
are equal to

1. $4x^3$

2. $4x^3$, $4x^3 + 5$, $4x^3 + 8$

3. $4x^3$, $4x^3 + 5$, $4x^3 - 8$

4. $4x^3$, $4x^3 - 5$, $4x^3 + 8$

4-10. All integrals of a given function
have the same constant of
integration.

1. True
2. False

4-11. If $\int f(x)\ dx = F(x) + C$, which
statement concerning C is TRUE?

1. It has only one value
2. It has only positive values
3. It has only negative values
4. It has an infinite number of
values

4-12. The statement "The integral of a
differential of a function is the
function plus a constant" is
illustrated by

1. $\int 2x\ dx = x^2 + C$

2. $\int dy = y + C$

3. $\int ax\ dx = ax^2 + C$

4. $\int a\ dx = ax + C$

4-13. Which expression is TRUE if g is
a constant and y is a variable?

1. $\int g\ dy = \int g \int dy$

2. $\int g\ dy = gy$

3. $\int g\ dy = g \int dy$

4. $\int g\ dy = y \int g$

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4-14. If $\int (dx + dy + dz)$

$= \int dx + \int dy + \int dz$, then

$\int dx + \int dy + \int dz$ is equal to

1. x - y + z
2. x + y + z + C
3. x + y + z
4. x + y - z

4-15. Evaluate the integral $\int x^2 \, dx$.

1. $2x$

2. $\dfrac{x^3}{3}$

3. $x^3 + C$

4. $\dfrac{x^3}{3} + C$

4-16. Which of the following integrals CANNOT be determined by applying

the rule, $\int u^n \, du = \dfrac{u^{n+1}}{n + 1} + C$?

1. $\int x^{-3} \, dx$

2. $\int x^{-1} \, dx$

3. $\int x^5 \, dx$

4. $\int x^{2/5} \, dx$

4-17. Evaluate the integral $\int \dfrac{x^7}{x^3} \, dx$.

1. $\dfrac{x^4}{4}$

2. $\dfrac{x^5}{5} + C$

3. $\dfrac{x^4}{4} + C$

4. $4x^2 + C$

4-18. Evaluate the integral

$\int 7(x^{1/2} + x) \, dx$.

1. $\dfrac{2}{3}x^{3/2} + \dfrac{x^2}{2} + C$

2. $\dfrac{14}{3}x^{3/2} + \dfrac{x^2}{2} + C$

3. $\dfrac{14}{3}x^{3/2} + \dfrac{7}{2}x^2 + C$

4. $\dfrac{2}{3}x^{3/2} + \dfrac{x^2}{2} + 7$

4-19. Given $\int dy = \int x^2 \, dx = \dfrac{x^3}{3} + C$,

what is the value of the constant of integration when x = 3 and y = 5?

1. It has an infinite number of values
2. -110/3
3. 14
4. -4

4-20. How do definite integrals differ from indefinite integrals?

1. The variable must be assigned a numerical value before the constant of integration can be found
2. The result of integration has a definite value
3. No constant of integration is needed
4. Both 2 and 3 above

4-21. $\int_{a}^{b} f(x) \, dx$ is an integral with

which one of the following characteristics?

1. b is the lower limit
2. a is the upper limit
3. b is more positive than a
4. a and b must both be positive

4-22. $\int_{5}^{10} f(x) \, dx = F(x) \Big|_{5}^{10}$ is the

same as

1. F(10) - F(5)
2. F(10) + F(5)
3. F5 - F(10)
4. F(5) - F(10)

4-23. The area bounded by the curve

$y = x^2 + 3$, the X axis, and x = 1 and x = 3 is

1. 8 2/3
2. 10 2/3
3. 12
4. 14 2/3

4-24. The area above the curve $y = -x^2$ but below the X axis and between x = 0 and x = 3 equals

1. -9
2. |-9|
3. 3
4. -3

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4-25. The area bounded by the curve
y = -x + 3, the X axis, x = 0,
and x = 6 equals (Hint: Be sure
to sketch the graph of the curve)

1. 0
2. 9
3. 18
4. 36

● The work, W, done in moving an
object from coordinate a to
coordinate b is given by

$$W = \int_a^b F(x)\ dx$$

where F(x) is the variable force applied
to the object.

This expression for work has
application to the force required in
stretching a spring. The force, F(x),
required to stretch an elastic spring
is directly proportional to the extension
of the spring (Hooke's Law) or

F(x) = kx

where k is the constant of proportionality.

EXAMPLE: If the natural length of
a spring is 18 inches and a force of 20
pounds will stretch the spring to 21
inches, find the amount of work done in
stretching the spring from a length of
20 inches to a length of 24 inches.

SOLUTION: Since it is given that
a force of 20 pounds will stretch the
spring 3 inches (21 inches - 18 inches)
or 1/4 foot, then solving F(x) = kx for
k gives

20 pounds = k(1/4 foot)
or
k = 80 pounds per foot
and
F(x) = 80x

The work done in stretching the spring
from 20 inches (20 - 18 = 2-inch
extension or 1/6-foot extension) to
24 inches (24 - 18 = 6-inch extension
or 1/2-foot extension) is then given by

$$W = \int_a^b F(x)\ dx$$

$$= \int_{1/6}^{1/2} 80x\ dx$$

$$= \frac{80x^2}{2}\ \Big|_{1/6}^{1/2}$$

$$= 40x^2\ \Big|_{1/6}^{1/2}$$

$$= 10 - 10/9$$

$$= 8\ 8/9\ \text{foot-pounds}$$

4-26. Find the work done in stretching
a 21-inch spring to 24 inches if
a force of 12 lbs is necessary
to stretch the spring from its
natural length of 21 inches to
21 1/2 inches.

1. 7 ft-lbs
2. 9 ft-lbs
3. 12 ft-lbs
4. 14 ft-lbs

● You may also use the expression

$$W = \int_a^b F(x)\ dx\ \text{in determining the}$$

work done in pumping water out of the
top of a tank. For a vertical
cylindrical tank with radius r feet and
height h feet, the weight of the water
remaining in the tank is equal to

$$F(x) = k\pi r^2(h - x)$$

where h - x is the height of the water
remaining in the tank and k = 62.5
(since water weighs about 62.5 pounds
per cubic foot). Therefore, the work
required to pump all of the water out
of the top of the cylindrical tank is
given by

$$W = \int_0^h 62.5\pi r^2(h - x)\ dx$$

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4-27. Calculate the work required to completely pump all of the water out of the top of a vertical cylindrical tank whose radius is 2 feet and height is 10 feet.

  1. 8,000π ft-lbs
  2. 9,500π ft-lbs
  3. 11,000π ft-lbs
  4. 12,500π ft-lbs

Learning Objective:

Simplify and solve integrals using formulas.

● The formulas below may be used for reference. In these formulas a, n, and C always represent a constant and u and v always represent a function of x.

Table 4A.--Formulas Involving Common Integral Forms

1. $\int du = u + C$

2. $\int a\, du = a \int du = au + C$

3. $\int (du + dv) = \int du + \int dv$
$$= u + v + C$$

4. $\int u^n\, du = \frac{u^{n+1}}{n+1} + C,\ n \neq -1$

5. $\int u^{-1}\, du = \int \frac{1}{u}\, du$
$$= \ln |u| + C,\ u \neq 0$$

6. $\int e^u\, du = e^u + C$

7. $\int a^u\, du = \frac{a^u}{\ln a} + C,\ a > 0$

8. $\int \sin u\, du = -\cos u + C$

9. $\int \cos u\, du = \sin u + C$

10. $\int \sec^2 u\, du = \tan u + C$

11. $\int \csc^2 u\, du = -\cot u + C$

12. $\int \sec u \tan u\, du = \sec u + C$

13. $\int \csc u \cot u\, du = -\csc u + C$

● In answering items 4-28 through 4-38, evaluate the given integral.

4-28. $\int 4x^3\, dx$ is

  1. $x^4 + C$

  2. $\frac{4}{3}x^4 + C$

  3. $12x^2 + C$

  4. $12x^4 + C$

4-29. $\int (5x^{3/2} - \pi x + 2x^{-5})\, dx$ is

  1. $\frac{25x^{5/2}}{2} - \frac{\pi}{2}x^2 - \frac{x^{-4}}{2} + C$

  2. $2x^{5/2} - \frac{\pi}{2}x^2 - \frac{x^{-6}}{3} + C$

  3. $2x^{5/2} - \frac{\pi}{2}x^2 - \frac{x^{-4}}{2} + C$

  4. $2x^{5/2} - \pi - \frac{x^{-4}}{2} + C$

4-30. $\int (x + 2)^2\, dx$ is

  1. $\frac{1}{3}(x + 2)^3 + C$

  2. $x^2 + 4x + 4$

  3. $2x + 4 + C$

  4. $2(x + 2)^3 + C$

4-31. $\int (x^3 - 3)^{1/2} x^2\, dx$ is

  1. $\frac{1}{2}(x^3 - 3)^{1/2} + C$

  2. $\frac{2}{9}(x^3 - 3)^{3/2} + C$

  3. $(x^3 - 3)^{1/2} x + C$

  4. $\frac{(x^4 - 3)^{3/2}}{4} + C$

4-32. $\int (x^2 - 6x + 2)^2 (2x - 6)\ dx$ is

1. $5x^5 - 2x^3 + x + C$

2. $2(x^2 - 6x + 2) + C$

3. $\frac{1}{3}(x^2 - 6x + 2)^3 + C$

4. $\frac{1}{6}(x^2 - 6x + 2)^3 (2x - 6)^2 + C$

4-33. $\int (x^2 - 8)(x^3 - 24x)^{-1/2}\ dx$ is

1. $\frac{1}{2}(x^2 - 8)^2 + C$

2. $-\frac{1}{2}(3x^2 - 24)^{-3/2} + C$

3. $x^{-3/2} - 24^{-1/2} + C$

4. $\frac{2}{3}(x^3 - 24x)^{1/2} + C$

4-34. $\int \dfrac{2x}{(1 + x^2)^3}\ dx$ is

1. $-1(1 + x^2)^2 + C$

2. $-\dfrac{1}{2(1 + x^2)^2} + C$

3. $\dfrac{1}{4(1 + x^2)^4} + C$

4. $\dfrac{1}{(1 + x^2)^3} + C$

4-35. $\int \dfrac{x - 1}{(2x - x^2)^{1/3}}\ dx$ is

1. $\dfrac{3(x - 1)^2}{8(2x - x^2)^{4/3}} + C$

2. $\frac{3}{2}(2x - x^2)^{2/3} + C$

3. $-\frac{3}{4}(2x - x^2)^{2/3} + C$

4. $-\frac{4}{3}(2x - x^2)^{4/3} + C$

4-36. $\int \dfrac{1}{x + 1}\ dx$ is

1. $\ln |x + 1| + C$

2. $\dfrac{1}{x + 1} + C$

3. $\dfrac{2}{(x + 1)^2} + C$

4. $2(x + 1)^2 + C$

4-37. $\int \dfrac{4}{x - 8}\ dx$ is

1. $\frac{1}{4} \ln |x - 8| + C$

2. $\frac{1}{2}(x - 8)^2 + C$

3. $\dfrac{8}{(x - 8)^2} + C$

4. $4 \ln |x - 8| + C$

4-38. $\int \dfrac{x}{2x^2 - 5}\ dx$ is

1. $4 \ln |2x^2 - 5| + C$

2. $\frac{1}{4} \ln |2x^2 - 5| + C$

3. $\dfrac{(2x^2 - 5)^2}{2} + C$

4. $\dfrac{2}{(2x^2 - 5)^2} + C$

4-39. When it is necessary to integrate a fractional function whose numerator contains a term to a higher power than any term in the denominator, the first step is to

1. apply the form $\int \frac{1}{u}\ du$

2. divide the denominator into the numerator

3. integrate each term in both numerator and denominator separately

4. divide each term of both numerator and denominator by the independent variable raised to the highest power contained in the numerator

• When the degree of the polynomial in the numerator is greater than or equal to the degree of the polynomial in the denominator, then division of a polynomial by a polynomial can be performed.

For example,

$(x^4 + x^2 + 2x - 80) \div (x + 3)$

could be determined as follows:

$$
\begin{array}{r}
x^3 - 3x^2 + 10x - 28 \\
x + 3 \overline{)\ x^4 + 0x^3 + x^2 + 2x - 80} \\
\underline{x^4 + 3x^3} \\
-3x^3 + x^2 \\
\underline{-3x^3 - 9x^2} \\
10x^2 + 2x \\
\underline{10x^2 + 30x} \\
-28x - 80 \\
\underline{-28x - 84} \\
4
\end{array}
$$

So,

$$
\frac{x^4 + x^2 + 2x - 80}{(x + 3)}
$$

$$
= x^3 - 3x^2 + 10x - 28 + \frac{4}{x + 3}
$$

• In answering items 4-40 through 4-54, evaluate the given integral.

4-40. $\int \frac{x^2}{x - 1}\ dx$ is

1. $2x + C$

2. $-\dfrac{x^3 - x^2 + 2x}{x^2 - 2x - 1} + C$

3. $\dfrac{(x + 1)^3}{3} + \ln |x - 1| + C$

4. $\frac{1}{2}x^2 + x + \ln |x - 1| + C$

4-41. $\int \dfrac{x^4 - 3x^2 - 4}{x + 2}\ dx$ is

1. $4x^3 - 6x + \ln |x + 2| + C$

2. $\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 + \ln \left| \dfrac{1}{(x + 2)^2} \right| + C$

3. $\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 - 2 \ln |x + 2| + C$

4. $\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 - 2x + C$

4-42. $\int e^x\ dx$ is

1. $e^x + C$

2. $\frac{1}{2}e^{x^2} + C$

3. $xe^{x-1} + C$

4. $\dfrac{e^x}{\ln x} + C$

4-43. $\int (x^2 - 1)e^{(3x-x^3)}\ dx$ is

1. $-\frac{1}{3} \ln |3x - x^3| + C$

2. $-\frac{1}{3}e^{(3x-x^3)} + C$

3. $e^{(3x-x^3)} + C$

4. $3e^{(3x-x^3)} + C$

4-44. $\int 2^x\ dx$ is

1. $2x + C$

2. $2^x \ln 2 + C$

3. $\dfrac{2^x}{\ln 2} + C$

4. $\dfrac{2^{x+1}}{x + 1} + C$

4-45. $\int 5^{(x/2-3)} \, dx$ is

    1. $\dfrac{2[5^{(x/2-3)}]}{\ln 5} + C$

    2. $\dfrac{5}{2}x - 15 + C$

    3. $\dfrac{5^{(x/2-3)}2}{\ln 5} + C$

    4. $\dfrac{1}{2}x - 3 \ln 5 + C$

4-46. $\int \sin \dfrac{x}{2} \, dx$ is

    1. $-\dfrac{1}{2} \cos \dfrac{x}{2} + C$

    2. $-2 \sin \dfrac{x}{2} + C$

    3. $-2 \cos \dfrac{x}{2} + C$

    4. $2 \tan \dfrac{x}{2} + C$

4-47. $\int x \cos 2x^2 \, dx$ is

    1. $-\sin 2x^2 + C$

    2. $\dfrac{1}{4} \sin 2x^2 + C$

    3. $\dfrac{x^2}{2} \cos \dfrac{2}{3}x^3 + C$

    4. $4 \sin 4x + C$

4-48. $\int (\cos 2x - \sin \dfrac{x}{3}) \, dx$ is

    1. $\sin 2x + \cos \dfrac{x}{3} + C$

    2. $6 \cos 2x \sin \dfrac{x}{3} + C$

    3. $\dfrac{1}{2} \sin 2x + 3 \cos \dfrac{x}{3} + C$

    4. $2 \sin 2x + \dfrac{x}{3} + C$

4-49. $\int \csc \dfrac{\theta}{2} \cot \dfrac{\theta}{2} \, d\theta$ is

    1. $2 \cos \dfrac{\theta}{2} \tan \dfrac{\theta}{2} + C$

    2. $2 \sec^2\left(\dfrac{\theta}{2}\right) + C$

    3. $-\dfrac{1}{2} \cot \dfrac{\theta}{2} + C$

    4. $-2 \csc \dfrac{\theta}{2} + C$

4-50. $\int \dfrac{\csc^2 \sqrt{x}}{\sqrt{x}} \, dx$ is

    1. $-\dfrac{1}{3} \cot^3(x^{1/2}) + C$

    2. $\dfrac{1}{2} \cot \sqrt{x} + C$

    3. $-\cot x^{-1/2} + C$

    4. $-2 \cot \sqrt{x} + C$

4-51. $\int x \sec (1 - x^2) \tan (1 - x^2) \, dx$ is

    1. $-\dfrac{1}{2} \sec (1 - x^2) + C$

    2. $-\sec 2x \tan 2x + C$

    3. $-\csc (1 - x^2) + C$

    4. $\tan (1 - x^2) + C$

4-52. $\int \sec^2 ax \, dx$ is

    1. $\dfrac{1}{3a} \sec^3 ax + C$

    2. $\dfrac{1}{a} \tan^2 ax + C$

    3. $\dfrac{1}{a} \tan ax + C$

    4. $a \tan ax + C$

4-53. $-6 \int \cos^2 x \sin x \, dx$ is

    1. $-6 \cos x + C$

    2. $-2 \cos x^2 + C$

    3. $2 \cos^3 x + C$

    4. $2 \sin^3 x + C$

4-54. $\int \sin^3\left(\frac{x}{3}\right) \cos \frac{x}{3} \, dx$ is

    1. $\frac{1}{12} \sin^4\left(\frac{x}{3}\right) + C$

    2. $\frac{3}{4} \sin^4\left(\frac{x}{3}\right) + C$

    3. $\frac{3}{4} \cos^4\left(\frac{x}{3}\right) + C$

    4. $\frac{1}{12} \cos^4\left(\frac{x}{3}\right) + C$